Efficient Numerical Implementation of a Vector Preisach Hysteresis Model for 3-D Finite Element Applications

Maxime Tousignant[1,2], Frédéric Sirois[1], Steven Dufour[1], and Gérard Meunier[2]

[1]Polytechnique Montreal, Canada, maxime.tousignant@polymtl.ca
[2]University Grenoble-Alpes, G2Elab, France

The design of electrical machines often requires the precise evaluation of the iron losses occurring in their core. This can be achieved by using the finite element technique with a hysteresis model to represent the magnetic properties of the iron core. The Preisach model is known in the literature to accurately model this behavior, however its high computational cost limits its use, especially in 3-D applications. This, paper presents a way to significantly improve the computational efficiency of the 3-D vector Preisach model by using a proper mathematical formulation and carefully implementing it.

***Index Terms*—Hysteresis, Finite elements analysis**

I. Introduction

THE STEADY gain in computational capabilities in the last decades has allowed for the numerical modelling of increasingly complex problems in electromagnetism by using the finite element (FE) method among others. In particular, the modelling of vector hysteresis to represent the behavior of magnetic materials in 3-D FE simulations has recently been achieved [1]. Most cases of vector hysteresis modelling reported in the literature usually use the Jiles-Artherton model because of its relative ease of implementation and good numerical performances. However, many comparative studies [2] have shown that the Preisach model (PM) provides more accurate results than the Jiles-Atherton model. Although these studies were conducted in the case of scalar hysteresis, a general vector extension of the Preisach model was introduced by Mayergoyz [3]. This vector extension of the PM using Mayergoyz's technique has already been successfully tested in 2-D FE [4], but its application in 3-D remains challenging because of its prohibitively high computation costs and memory requirements. This issue is the focus of this paper.

We first present a mathematical formulation of the vector PM based on Mayergoyz's method with a sight modification that allows the material to be initialized in a perfectly demagnetized state. Then, some considerations regarding the efficient numerical implementation of the model are discussed.

II. Preisach Model: Mathematical Formulation

The vector extension of the PM proposed by Mayergoyz consists in calculating the magnetization vector $\mathbf{M}$ as a sum of numerous scalar PM oriented in different directions. Therefore, it is essential to begin with the presentation of the scalar PM since it is the basis to construct the vector extension.

A. The Scalar Preisach Model

The classical scalar PM relates the magnetization M to the magnetic field H. It can be formulated using the Everett

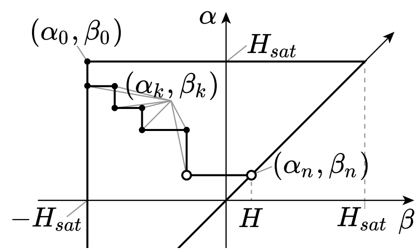

Fig. 1. The (α_k, β_k) coordinates associated with the magnetic field history define a staircase shaped boundary in the $\alpha\beta$ domain, separating the positive and negative contributions to the magnetization.

function as

$$M(H) = -E(\alpha_0, \beta_0) + 2\sum_{k=1}^{n} \left[E(\alpha_k, \beta_{k-1}) - E(\alpha_k, \beta_k)\right] , \tag{1}$$

where $(\alpha_0, \beta_0) = (H_{sat}, -H_{sat})$, $(\alpha_n, \beta_n) = (H, H)$ and (α_k, β_k) is associated with the k^{th} local extremum of decreasing amplitude in the magnetic field history (see Fig. 1). The magnetic field at saturation H_{sat} is the value of magnetic field where the two branches of the major hysteresis loop merge.

One issue with this formulation in the context of FE modelling is that, in order to initialize a magnetic material in its demagnetized state (i.e. $M(H) = 0$), one would in theory need an infinite number of points (α_k, β_k) along the $\alpha = -\beta$ diagonal. In practice one would typically approximate the demagnetized state with a large number (α_k, β_k) points in the history, which is not only inaccurate, but also very costly in terms of memory. The solution we propose to fix this problem is to redefine (α_0, β_0) as

$$(\alpha_0, \beta_0) = (H_{max}, -H_{max}) \tag{2}$$

where

$$H_{max} = \max_{t' \in [0,t]} |H(t')| \, . \tag{3}$$

That way, we can initialize the material in a perfectly demagnetized state with a single point in the history $(\alpha_0, \beta_0) = (0, 0)$, while effectively assuming an infinite number of virtual points along the $\alpha = -\beta$ diagonal.

B. The Vector Preisach Model

The 3-D vector PM proposed by Mayergoyz in [3] consist in integrating the contributions of an infinite number of scalar PM spanning every possible orientation.

In spherical coordinates, for an orientation defined by the unit vector $\mathbf{e}_{\theta,\varphi}$, the projection of the magnetic field vector onto this orientation is

$$H_{\theta,\varphi} = \mathbf{e}_{\theta,\varphi} \cdot \mathbf{H}\,. \tag{4}$$

The contribution from this orientation, noted $M_{\theta,\varphi}$, is calculated using (1) with $H_{\theta,\varphi}$ as input. Then, by integrating the contribution from every orientation, we get

$$\mathbf{M}(\mathbf{H}) = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \mathbf{e}_{\theta,\varphi}\, M_{\theta,\varphi}(H_{\theta,\varphi})\, \sin\theta\, d\theta\, d\varphi\,. \tag{5}$$

III. Numerical Implementation

The numerical implementation of the 3-D vector PM comes down to the evaluation of (5), which requires the computation of a certain number of scalar PM distributed in different orientations using (1). Therefore, in order to optimize the numerical performance of the vector PM, we must first minimize the number of orientations (of scalar PM) that need to be computed, and then, optimize the numerical performance of the scalar PM.

Firstly, we discretize the integral over the surface of the unit hemisphere in (5) using the Lebedev quadrature [5]. Thus, we can approximate the vector PM as

$$\mathbf{M}(\mathbf{H}) \approx \frac{1}{2\pi}\sum_{i=1}^{N_L} w_i\, \mathbf{e}_{\theta_i,\varphi_i}\, M_{\theta_i,\varphi_i}(H_{\theta_i,\varphi_i})\,, \tag{6}$$

where N_L is the number of points in the quadrature and therefore the number of scalar PM that must be computed. There is obviously a trade off to be made here between computational speed and precision, however we found that using a Lebedev quadrature of 7$^{\text{th}}$ order, for which $N_L = 13$, leads to good results.

In the case where the Everett function is evaluated using an analytic function instead of interpolating experimental data, we can extract the reversible component of the magnetization from the Everett function. By doing so, the reversible component is only computed once instead of 13 times, and it is not affected by the limited precision of the Lebedev quadrature.

Secondly, regarding the numerical performance of the scalar PM, other than implementing the perfectly demagnetized formulation (see Sec. II-A), we can also optimize the computation of (1) by reducing the number of evaluations of the Everett function. Indeed, due to the staircase shape of the Preisach boundary, we notice that for roughly half the terms, $\beta_{k-1} = \beta_k$ and therefore $[E(\alpha_k, \beta_{k-1}) - E(\alpha_k, \beta_k)] = 0$. Furthermore, we can take advantage of the fact that at most time steps, only the coordinates $(\alpha_{n-1}, \beta_{n-1})$ and (α_n, β_n) are modified in the history (white dots in Fig. 1). Thus, storing the value of the partial sum in (1) up to $n-2$ avoids a lot of unnecessary computations.

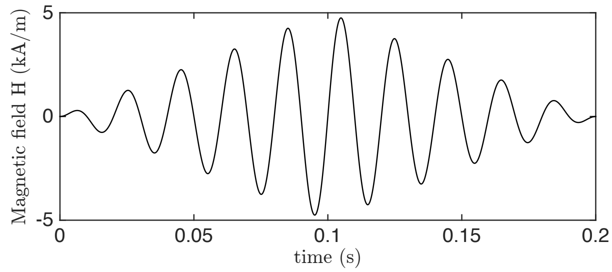

Fig. 2. The magnetic field gradually saturates the material in the first half of the simulation and demagnetizes it in the second half.

IV. Results

To quantify the speed-up provided by the various implementation techniques proposed, we implemented five different variations of the vector PM. Every model was initialized in the demagnetized state, using either a regular staircase Preisach boundary along the $\alpha = -\beta$ diagonal composed of 16 points (models A, B, and C) or the proposed perfectly demagnetized formulation (models D and E). In every case, the Lebedev quadrature of 7$^{\text{th}}$ order with 13 orientations was chosen. The Everett function was computed using an analytical formula. We simulated the response of the models subject to an alternating magnetic field of varying amplitude display in Fig. 2. The results for this test are presented in table I.

TABLE I
Gain in numerical performance

	Model description	relative cpu time	memory* (bytes)
A)	Basic implementation (Reference)	1.000	403
B)	Reversible-irreversible seperation	0.556	403
C)	Efficient summation in Eq. (1)	0.098	416
D)	Perfect demagnetization	0.252	39
E)	Combination of B, C and D	0.047	52

* Memory per calculation node at initialization

As the results indicate, an efficient numerical implementation of the vector PM allows for significant speed-ups and memory reductions. Indeed, the computation time for the final model was less than 5% than that of the original one, and the memory required to initialize the material in the demagnetized state was reduced by an order of magnitude.

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